

# Exploring CP Violation and $\eta$ – $\eta'$ Mixing with the $B_{s,d}^0 \rightarrow J/\psi \eta^{(\prime)}$ Systems

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## Abstract

The  $B_{s,d}^0 \rightarrow J/\psi \eta^{(\prime)}$  decays provide new terrain for exploring CP violation. After briefly discussing  $\eta$ – $\eta'$  mixing, we analyse the effective lifetimes and CP-violating observables of the  $B_s$  channels, which allow us to probe New-Physics effects in  $B_s^0$ – $\bar{B}_s^0$  mixing. We have a critical look at these observables and show how hadronic corrections can be controlled by means of the  $B_d$  decays. Using measurements of the  $B_{s,d}^0 \rightarrow J/\psi \eta^{(\prime)}$  branching ratios by the Belle collaboration, we discuss tests of the  $SU(3)_F$  flavour symmetry of strong interactions, obtain the first constraints on the hadronic parameters entering the  $B_{s,d}^0 \rightarrow J/\psi \eta$  system, and predict the  $B_d^0 \rightarrow J/\psi \eta'$  branching ratio at the  $5 \times 10^{-6}$  level. Furthermore, we present strategies for the determination of the  $\eta$ – $\eta'$  mixing parameters from the  $B_{s,d}^0 \rightarrow J/\psi \eta^{(\prime)}$  observables. We also observe that the  $B_{s,d}^0 \rightarrow J/\psi \eta$  and  $B_{s,d}^0 \rightarrow J/\psi \eta'$  decays are – from a formal point of view – analogous to the quark–antiquark and tetraquark descriptions of the  $f_0(980)$  in the  $B_{s,d}^0 \rightarrow J/\psi f_0(980)$  channels, respectively.



# 1 Introduction

With the Large Hadron Collider (LHC) at CERN collecting plenty of data, tests of the Standard Model (SM) have entered a new era. Concerning the exploration of the quark-flavour sector, the study of CP violation in  $B_s$ -meson decays at the LHCb experiment is one of the most exciting aspects of this endeavor. In addition to various, by now “standard”,  $B_s$  decays [1], another interesting probe is offered by the  $B_s^0 \rightarrow J/\psi\eta^{(\prime)}$  channels. In these decays, New Physics (NP) can enter through CP-violating contributions to  $B_s^0$ – $\bar{B}_s^0$  mixing, which is a strongly suppressed loop phenomenon in the SM (see, for instance, Refs. [2] and references therein).

In Ref. [3], a determination of the angle  $\gamma$  of the Unitarity Triangle (UT) of the Cabibbo–Kobayashi–Maskawa (CKM) matrix was proposed that relates the  $B_d^0 \rightarrow J/\psi\eta^{(\prime)}$  decays to  $B_s^0 \rightarrow J/\psi\eta^{(\prime)}$  through the  $SU(3)_F$  flavour symmetry of strong interactions. This method is a variant of the  $B_{s,d}^0 \rightarrow J/\psi K_S$  strategy proposed in Ref. [4]. As was recently shown [5], the extraction of  $\gamma$  from the  $B_{s,d}^0 \rightarrow J/\psi K_S$  system is possible at LHCb but cannot compete with other strategies [1]; the situation for  $B_{d,s}^0 \rightarrow J/\psi\eta^{(\prime)}$  looks even more challenging. However, the  $B_s^0 \rightarrow J/\psi K_S$  mode will still play an important role at LHCb as a “control channel”, allowing us to take hadronic SM corrections in the extraction of the UT angle  $\beta$  from the CP violation in  $B_d^0 \rightarrow J/\psi K_S$  into account [4, 5].

In the present paper, we shall follow a similar avenue, assuming that  $\gamma$  will be measured at LHCb with a precision of a few degrees by means of the corresponding benchmark decays in the next couple of years [1]. We will then use the  $B_d^0 \rightarrow J/\psi\eta^{(\prime)}$  decays to control the hadronic corrections to observables of the  $B_s^0 \rightarrow J/\psi\eta^{(\prime)}$  modes, which are sensitive to NP effects in  $B_s^0$ – $\bar{B}_s^0$  mixing. To be specific, we will study the effective lifetimes and CP-violating rate asymmetries of the  $B_s^0 \rightarrow J/\psi\eta^{(\prime)}$  decays. In contrast to the rate asymmetries, the lifetime analysis utilizes the sizable width difference  $\Delta\Gamma_s$  between the  $B_s$  mass eigenstates and requires only untagged  $B_s$  data samples. Here one does not distinguish between initially present  $B_s^0$  or  $\bar{B}_s^0$  mesons, which is advantageous from an experimental point of view.

The power of the  $B_s^0 \rightarrow J/\psi\eta^{(\prime)}$  observables to reveal CP-violating NP contributions to  $B_s^0$ – $\bar{B}_s^0$  mixing is limited by doubly Cabibbo-suppressed contributions to the decay amplitudes. We will explore the impact of the relevant non-perturbative parameters, which cannot be calculated reliably in QCD, and discuss how they can be determined from  $B_d^0 \rightarrow J/\psi\eta^{(\prime)}$  data through the  $SU(3)_F$  flavour symmetry of strong interactions. Thanks to their different CKM amplitude structure, the hadronic parameters are not doubly Cabibbo-suppressed in these channels, thereby leading to significant effects in the corresponding observables.

Using  $B_{s,d}^0 \rightarrow J/\psi\eta$  branching ratio measurements by the Belle collaboration, we introduce quantities to probe  $SU(3)_F$ -breaking effects, obtain first constraints on the relevant penguin parameters, and discuss strategies to extract the  $\eta$ – $\eta'$  mixing parameters. We also point out that the  $B_{s,d}^0 \rightarrow J/\psi\eta$  and  $B_{s,d}^0 \rightarrow J/\psi\eta'$  decays are – from a formal point of view – analogous to the quark–antiquark and tetraquark descriptions of the scalar  $f_0(980)$  state in  $B_s^0 \rightarrow J/\psi f_0(980)$ , respectively [6]. For simplicity, we shall from here on abbreviate the  $f_0(980)$  as  $f_0$ .

Unfortunately, the experimental analyses of the  $B_s^0 \rightarrow J/\psi\eta^{(\prime)}$  modes are complicated

by the reconstruction of the  $\eta^{(\prime)}$  decays. The most prominent channels are  $\eta \rightarrow 2\gamma$ ,  $3\pi^0$ ,  $\pi^+\pi^-\pi^0$ ,  $\pi^+\pi^-\gamma$  and  $\eta' \rightarrow \pi^+\pi^-\eta$ ,  $\rho^0\gamma$ ,  $\pi^0\pi^0\eta$  [7], which have challenging signatures for studies at hadron colliders. At the future  $e^+e^-$  SuperKEKB and SuperB projects the prospects of measuring these decays may be more promising.

The outline is as follows: in Section 2, we give a brief overview of  $\eta$ - $\eta'$  mixing and discuss how it is implemented in the  $B_s^0 \rightarrow J/\psi\eta^{(\prime)}$  decay amplitudes. In Section 3, we turn to the effective lifetimes and the CP-violating observables of the  $B_s^0 \rightarrow J/\psi\eta^{(\prime)}$  transitions. In Section 4, we focus on the  $B_d^0 \rightarrow J/\psi\eta^{(\prime)}$  decays and their role as control channels. Finally, we discuss determinations of the  $\eta$ - $\eta'$  mixing parameters from the  $B_{s,d}^0 \rightarrow J/\psi\eta^{(\prime)}$  branching ratios in Section 5, and summarize our conclusions in Section 6.

## 2 The $B_s^0 \rightarrow J/\psi\eta^{(\prime)}$ Decay Amplitudes

Before focusing on the  $B_s^0 \rightarrow J/\psi\eta^{(\prime)}$  decays, we will first give a brief overview of  $\eta$ - $\eta'$  mixing. The physical  $|\eta\rangle$  and  $|\eta'\rangle$  states are mixtures of the octet and singlet states  $|\eta_8\rangle$  and  $|\eta_1\rangle$ , respectively, and can be written as follows [7]:

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_P & -\sin\theta_P \\ \sin\theta_P & \cos\theta_P \end{pmatrix} \cdot \begin{pmatrix} |\eta_8\rangle \\ |\eta_1\rangle \end{pmatrix}, \quad (1)$$

where

$$|\eta_8\rangle = \frac{1}{\sqrt{6}} (|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle), \quad |\eta_1\rangle = \frac{1}{\sqrt{3}} (|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle). \quad (2)$$

The mixing between the octet and singlet states is a manifestation of the breaking of the  $SU(3)_F$  flavour symmetry of strong interactions. Alternatively,  $\eta$ - $\eta'$  mixing can be described in terms of the isospin singlet states

$$|\eta_q\rangle \equiv \frac{1}{\sqrt{2}} (|u\bar{u}\rangle + |d\bar{d}\rangle), \quad |\eta_s\rangle \equiv |s\bar{s}\rangle. \quad (3)$$

By also taking the possible mixing with a purely gluonic component  $|gg\rangle$  into account, we can write the following expressions (for a recent detailed discussion, see Ref. [8]):

$$|\eta\rangle = \cos\phi_P |\eta_q\rangle - \sin\phi_P |\eta_s\rangle, \quad (4)$$

$$|\eta'\rangle = \cos\phi_G \sin\phi_P |\eta_q\rangle + \cos\phi_G \cos\phi_P |\eta_s\rangle + \sin\phi_G |gg\rangle. \quad (5)$$

Here it has been assumed, for simplicity, that the heavier  $\eta'$  contains a larger gluonic admixture than the lighter  $\eta$  and that the coupling of the latter state to  $|gg\rangle$  is negligible. Estimates give  $\sin^2\phi_G \sim 0.1$  [9], i.e.  $|\phi_G| \sim 20^\circ$ , which indicates that the impact of this contribution is suppressed.

The mixing angle  $\phi_P$  is still subject of ongoing studies, using data for processes such as  $D_s^+ \rightarrow \eta^{(\prime)}\ell^+\nu_\ell$  decays and the two-photon width of the  $\eta^{(\prime)}$  mesons (see Ref. [8] and references therein). The full spectrum of results correspond to  $30^\circ \lesssim \phi_P \lesssim 45^\circ$ , with the majority of analyses converging at values of  $\phi_P$  around  $40^\circ$ . Consequently, the relations

$$\cos\phi_P \approx \sqrt{\frac{2}{3}}, \quad \sin\phi_P \approx \sqrt{\frac{1}{3}}, \quad (6)$$

where the numerical values correspond to  $\phi_P = 35^\circ$ , are affected by uncertainties of  $\mathcal{O}(20\%)$ . These approximate relations result in the simple expressions

$$|\eta\rangle \approx \frac{1}{\sqrt{3}} (|u\bar{u}\rangle + |d\bar{d}\rangle - |s\bar{s}\rangle) \quad (7)$$

$$|\eta'\rangle \approx \frac{1}{\sqrt{6}} (|u\bar{u}\rangle + |d\bar{d}\rangle + 2|s\bar{s}\rangle) \cos \phi_G + \sin \phi_G |gg\rangle, \quad (8)$$

which are useful for  $SU(3)_F$  analyses of non-leptonic  $B$ -meson decays with  $\eta^{(\prime)}$  mesons in the final states [10,11]. In our study we shall follow a similar conceptual avenue, keeping, however,  $\phi_P$  as a free parameter.

The  $B_s^0 \rightarrow J/\psi\eta$  mode has dynamics very similar to  $B_s^0 \rightarrow J/\psi f_0$  with a quark–antiquark description assumed for the  $f_0$  [6]. In particular, the decay topologies are the same, and to obtain the transition amplitude we only need to make the following substitutions in the relevant formulae:

$$\cos \varphi_M \rightarrow -\sin \phi_P, \quad \sin \varphi_M \rightarrow \cos \phi_P, \quad (9)$$

i.e.  $\varphi_M$  introduced in Ref. [6] should be replaced by  $\phi_P + 90^\circ$ . The  $f_0$  mixing angle corresponding to (6),  $\varphi_M \approx 125^\circ$ , is consistent with phenomenological analyses of the scalar  $f_0$  state in the quark–antiquark picture (see Ref. [6] and references therein).

Using the unitarity of the CKM matrix, the decay amplitude can be written as

$$A(B_s^0 \rightarrow J/\psi\eta) = \left(1 - \frac{\lambda^2}{2}\right) \mathcal{A}_\eta [1 + \epsilon b_\eta e^{i\vartheta_\eta} e^{i\gamma}] \quad (10)$$

with

$$\mathcal{A}_\eta = -\lambda^2 A \left[ \sin \phi_P \left\{ \tilde{A}_T^{(c)} + \tilde{A}_P^{(ct)} + \tilde{A}_E^{(c)} + \tilde{A}_{PA}^{(ct)} \right\} - \frac{1}{\sqrt{2}} \cos \phi_P \left\{ 2\tilde{A}_E^{(c)} + 2\tilde{A}_{PA}^{(ct)} \right\} \right] \quad (11)$$

and

$$b_\eta e^{i\vartheta_\eta} = R_b \left[ \frac{\sin \phi_P \left\{ \tilde{A}_P^{(ut)} + \tilde{A}_{PA}^{(ut)} \right\} - \frac{1}{\sqrt{2}} \cos \phi_P \left\{ \tilde{A}_E^{(u)} + 2\tilde{A}_{PA}^{(ut)} \right\}}{\sin \phi_P \left\{ \tilde{A}_T^{(c)} + \tilde{A}_P^{(ct)} + \tilde{A}_E^{(c)} + \tilde{A}_{PA}^{(ct)} \right\} - \frac{1}{\sqrt{2}} \cos \phi_P \left\{ 2\tilde{A}_E^{(c)} + 2\tilde{A}_{PA}^{(ct)} \right\}} \right], \quad (12)$$

where we have used the isospin and  $SU(3)_F$  flavour symmetries of strong interactions to identify certain amplitudes and hereby simplify the expressions. In analogy to the discussion in Ref. [6],  $\mathcal{A}_\eta$  and  $b_\eta e^{i\vartheta_\eta}$  are CP-conserving strong parameters, which encode the hadron dynamics of the  $B_s^0 \rightarrow J/\psi\eta$  decay; the labels T, P, E and PA refer to tree, penguin, exchange and penguin annihilation topologies, respectively. As usual,  $\lambda \equiv |V_{us}| = 0.2252 \pm 0.0009$  denotes the Wolfenstein parameter [7], while

$$\epsilon \equiv \frac{\lambda^2}{1 - \lambda^2} = 0.0534 \pm 0.0005, \quad A \equiv \frac{|V_{cb}|}{\lambda^2} \sim 0.8, \quad R_b \equiv \left(1 - \frac{\lambda^2}{2}\right) \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right| \sim 0.5. \quad (13)$$

Using (6), we obtain the following simplified expressions:

$$\mathcal{A}_\eta \approx -\lambda^2 A \sqrt{\frac{1}{3}} \left[ \tilde{A}_T^{(c)} + \tilde{A}_P^{(ct)} - \tilde{A}_E^{(c)} - \tilde{A}_{PA}^{(ct)} \right], \quad (14)$$

$$b_\eta e^{i\vartheta_\eta} \approx R_b \left[ \frac{\tilde{A}_P^{(ut)} - \tilde{A}_E^{(u)} - \tilde{A}_{PA}^{(ut)}}{\tilde{A}_T^{(c)} + \tilde{A}_P^{(ct)} - \tilde{A}_E^{(c)} - \tilde{A}_{PA}^{(ct)}} \right]. \quad (15)$$

The  $B_s^0 \rightarrow J/\psi \eta'$  amplitude takes the same form as (10). The corresponding parameters  $\mathcal{A}_{\eta'}$  and  $b_{\eta'} e^{i\vartheta_{\eta'}}$  can be obtained from the expressions in Ref. [6] by making the simple substitution  $\varphi_M \rightarrow \phi_P$ . We observe that the relations in (6) give a structure of the  $B_s^0 \rightarrow J/\psi \eta'$  amplitude that is analogous to that for  $B_s^0 \rightarrow J/\psi f_0$  with the tetraquark interpretation of the  $f_0$ . In this case, there is an additional topology that is specific to the  $f_0$  tetraquark state. On the other hand, we have an additional contribution to  $B_s^0 \rightarrow J/\psi \eta'$  from the gluonic component of the  $\eta'$ . Using (6), we arrive at

$$\mathcal{A}_{\eta'} \approx \lambda^2 A \sqrt{\frac{2}{3}} \left[ \tilde{A}_T^{(c)} + \tilde{A}_P^{(ct)} + 2\tilde{A}_E^{(c)} + 2\tilde{A}_{PA}^{(ct)} + \sqrt{\frac{3}{2}} \left( \tilde{A}_{E,gg}^{(c)} + \tilde{A}_{PA,gg}^{(ct)} \right) \tan \phi_G \right] \cos \phi_G \quad (16)$$

$$b_{\eta'} e^{i\vartheta_{\eta'}} \approx R_b \left[ \frac{\tilde{A}_P^{(ut)} + \frac{1}{2}\tilde{A}_E^{(u)} + 2\tilde{A}_{PA}^{(ut)} + \sqrt{\frac{3}{2}}\tilde{A}_{PA,gg}^{(ut)} \tan \phi_G}{\tilde{A}_T^{(c)} + \tilde{A}_P^{(ct)} + 2\tilde{A}_E^{(c)} + 2\tilde{A}_{PA}^{(ct)} + \sqrt{\frac{3}{2}} \left( \tilde{A}_{E,gg}^{(c)} + \tilde{A}_{PA,gg}^{(ct)} \right) \tan \phi_G} \right], \quad (17)$$

where  $\tilde{A}_{\text{topology},gg}^{(q)}$  denotes a strong amplitude originating from the  $|gg\rangle$  admixture. As indicated, the gluonic component can only contribute through exchange and penguin annihilation topologies, which are expected to be small in comparison to the tree and penguin topologies, respectively [6]. A further suppression comes from  $\tan^2 \phi_G \sim 0.1$ . It is interesting to note in passing that the dynamics are different in  $B \rightarrow K \eta'$  decays, where a gluonic component of the  $\eta'$  can contribute in the leading penguin topologies.

### 3 The $B_s^0 \rightarrow J/\psi \eta^{(\prime)}$ Observables

The Belle collaboration reported the observation of  $B_s^0 \rightarrow J/\psi \eta$  and evidence for the  $B_s^0 \rightarrow J/\psi \eta'$  decay in 2009, with the following branching ratio measurements [12]:

$$\text{BR}(B_s^0 \rightarrow J/\psi \eta) = [3.32 \pm 0.87 (\text{stat.})_{-0.28}^{+0.32} (\text{syst.}) \pm 0.42 (f_s)] \times 10^{-4} \quad (18)$$

$$\text{BR}(B_s^0 \rightarrow J/\psi \eta') = [3.1 \pm 1.2 (\text{stat.})_{-0.6}^{+0.5} (\text{syst.}) \pm 0.38 (f_s)] \times 10^{-4}. \quad (19)$$

Here the latter errors refer to the  $B_s$  fragmentation function  $f_s$ .

Using the  $SU(3)_F$  flavour symmetry, these branching ratios can be related to that of  $B_d^0 \rightarrow J/\psi K^0$ . Taking factorizable  $SU(3)_F$ -breaking corrections into account yields

$$\frac{\text{BR}(B_s^0 \rightarrow J/\psi \eta^{(\prime)})}{\text{BR}(B_d^0 \rightarrow J/\psi K^0)} \Big|_{\text{fact.}} = \frac{\tau_{B_s^0}}{\tau_{B_d^0}} \left[ \frac{M_{B_s^0} \Phi_s^{\eta^{(\prime)}}}{M_{B_d^0} \Phi_d^{K^0}} \right]^3 \left[ \frac{F_1^{B_s^0 \eta^{(\prime)}}(M_{J/\psi}^2)}{F_1^{B_d^0 K^0}(M_{J/\psi}^2)} \right]^2, \quad (20)$$

where the  $\tau_{B_q^0}$  and  $M_{B_q^0}$  are the  $B_q^0$  lifetimes and masses, respectively,

$$\Phi_q^P \equiv \sqrt{\left[ 1 - \left( \frac{M_P + M_{J/\psi}}{M_{B_q}} \right)^2 \right] \left[ 1 - \left( \frac{M_P - M_{J/\psi}}{M_{B_q}} \right)^2 \right]} \quad (21)$$

denotes the phase-space factor for  $B_q^0 \rightarrow J/\psi P$  decays, and the  $F_1^{B_q^0 P}(M_{J/\psi}^2)$  are hadronic form factors of quark currents (for a detailed discussion, see Ref. [6]). These relations have been used previously to predict the  $B_s^0 \rightarrow J/\psi \eta^{(\prime)}$  branching ratios [3, 13].

We advocate to use the measured values to probe non-factorizable  $SU(3)_F$ -breaking corrections. To this end we define the quantities

$$K_{SU(3)}^{\eta^{(\prime)}} \equiv \frac{\tau_{B_d^0}}{\tau_{B_s^0}} \left[ \frac{M_{B_d^0} \Phi_d^{K^0}}{M_{B_s^0} \Phi_s^{\eta^{(\prime)}}} \right]^3 \left[ \frac{F_1^{B_d^0 K^0}(M_{J/\psi}^2)}{F_1^{B_s^0 \eta^{(\prime)}}(M_{J/\psi}^2)} \right]^2 \frac{\text{BR}(B_s^0 \rightarrow J/\psi \eta^{(\prime)})}{\text{BR}(B_d^0 \rightarrow J/\psi K^0)}, \quad (22)$$

where  $K_{SU(3)}^{\eta^{(\prime)}} = 1$  in the case of vanishing non-factorizable  $SU(3)_F$ -breaking corrections. Since non-perturbative calculations of the  $F_1^{B_s^0 \eta^{(\prime)}}(M_{J/\psi}^2)$  form factors are not yet available, we project out on the  $|\eta_s\rangle$  component in (4) and write

$$F_1^{B_s^0 \eta}(M_{J/\psi}^2) = -\sin \phi_P F_1^{B_d^0 K^0}(M_{J/\psi}^2) \quad (23)$$

$$F_1^{B_s^0 \eta'}(M_{J/\psi}^2) = \cos \phi_G \cos \phi_P F_1^{B_d^0 K^0}(M_{J/\psi}^2), \quad (24)$$

where we neglect  $SU(3)_F$ -breaking corrections originating from the down and strange spectator quarks. Using  $\text{BR}(B_d^0 \rightarrow J/\psi K^0) = (8.71 \pm 0.32) \times 10^{-4}$  [7] then yields

$$K_{SU(3)}^\eta = \left[ \frac{\sin 40^\circ}{\sin \phi_P} \right]^2 \times (0.87 \pm 0.27), \quad K_{SU(3)}^{\eta'} = \left[ \frac{\cos 20^\circ}{\cos \phi_G} \right]^2 \left[ \frac{\cos 40^\circ}{\cos \phi_P} \right]^2 \times (0.8 \pm 0.4). \quad (25)$$

These numbers do not indicate any anomalous behaviour, although the currently large errors preclude us from drawing stronger conclusions. In Section 5, we will return to the  $B_s^0 \rightarrow J/\psi \eta^{(\prime)}$  branching ratios, using them to extract the mixing angles  $\phi_P$  and  $\phi_G$ .

A simple observable that is offered by the  $B_s^0 \rightarrow J/\psi \eta^{(\prime)}$  decays is their effective lifetime, which is defined through the time expectation value [14]

$$\tau_{J/\psi \eta^{(\prime)}} \equiv \frac{\int_0^\infty t \langle \Gamma(B_s(t) \rightarrow J/\psi \eta^{(\prime)}) \rangle dt}{\int_0^\infty \langle \Gamma(B_s(t) \rightarrow J/\psi \eta^{(\prime)}) \rangle dt} \quad (26)$$

of the untagged rate

$$\begin{aligned} \langle \Gamma(B_s(t) \rightarrow J/\psi \eta^{(\prime)}) \rangle &\equiv \Gamma(B_s^0(t) \rightarrow J/\psi \eta^{(\prime)}) + \Gamma(\bar{B}_s^0(t) \rightarrow J/\psi \eta^{(\prime)}) \\ &= R_H^{J/\psi \eta^{(\prime)}} e^{-\Gamma_H^{(s)} t} + R_L^{J/\psi \eta^{(\prime)}} e^{-\Gamma_L^{(s)} t} \\ &\propto e^{-\Gamma_s t} \left[ \cosh \left( \frac{\Delta \Gamma_s t}{2} \right) + \mathcal{A}_{\Delta \Gamma}^{J/\psi \eta^{(\prime)}} \sinh \left( \frac{\Delta \Gamma_s t}{2} \right) \right], \end{aligned} \quad (27)$$

where L and H denote the light and heavy  $B_s$  mass eigenstates, respectively,  $\Delta \Gamma_s \equiv \Gamma_L^{(s)} - \Gamma_H^{(s)}$  and  $\Gamma_s \equiv (\Gamma_L^{(s)} + \Gamma_H^{(s)})/2 = \tau_{B_s}^{-1}$ . This lifetime is equivalent to that resulting from a fit of the two exponentials in (27) to a single exponential [15]. The effective lifetime can thus be expressed as

$$\frac{\tau_{J/\psi \eta^{(\prime)}}}{\tau_{B_s}} = \frac{1}{1 - y_s^2} \left[ \frac{1 + 2 \mathcal{A}_{\Delta \Gamma}^{J/\psi \eta^{(\prime)}} y_s + y_s^2}{1 + \mathcal{A}_{\Delta \Gamma}^{J/\psi \eta^{(\prime)}} y_s} \right], \quad (28)$$

where  $y_s \equiv \Delta\Gamma_s/(2\Gamma_s)$ . The most recent update for the theoretical calculation of the  $B_s$  width difference is given as follows [16]:

$$\frac{\Delta\Gamma_s^{\text{Th}}}{\Gamma_s} = 0.133 \pm 0.032. \quad (29)$$

In Ref. [6], the evaluation of  $\mathcal{A}_{\Delta\Gamma}$  has been discussed in detail for the  $B_s^0 \rightarrow J/\psi f_0$  channel, which has a CP-odd final state and offers an interesting probe of CP violation [17]. Since the  $\eta^{(\prime)}$  are pseudoscalar mesons with quantum numbers  $J^{PC} = 0^{-+}$ , the final states of  $B_{s,d}^0 \rightarrow J/\psi \eta^{(\prime)}$  are CP even. This sign difference results in

$$\mathcal{A}_{\Delta\Gamma}^{J/\psi \eta^{(\prime)}} = -\sqrt{1 - C_{J/\psi \eta^{(\prime)}}^2} \cos(\phi_s + \Delta\phi_{J/\psi \eta^{(\prime)}}). \quad (30)$$

Here  $C_{J/\psi \eta^{(\prime)}}$  describes direct CP violation, whereas

$$\phi_s \equiv \phi_s^{\text{SM}} + \phi_s^{\text{NP}} \quad (31)$$

denotes the  $B_s^0$ - $\bar{B}_s^0$  mixing phase, where [18]

$$\phi_s^{\text{SM}} \equiv -2\beta_s = -(2.08 \pm 0.09)^\circ \quad (32)$$

and  $\phi_s^{\text{NP}}$  are the SM and NP pieces, respectively. The quantity  $\Delta\phi_{J/\psi \eta^{(\prime)}}$  is a hadronic phase shift, which can be obtained from

$$\tan \Delta\phi_{J/\psi \eta^{(\prime)}} = \frac{2\epsilon b_{\eta^{(\prime)}} \cos \vartheta_{\eta^{(\prime)}} \sin \gamma + \epsilon^2 b_{\eta^{(\prime)}}^2 \sin 2\gamma}{1 + 2\epsilon b_{\eta^{(\prime)}} \cos \vartheta_{\eta^{(\prime)}} \cos \gamma + \epsilon^2 b_{\eta^{(\prime)}}^2 \cos 2\gamma}. \quad (33)$$

We observe that the hadronic parameters, which are poorly known, enter  $\Delta\phi_{J/\psi \eta^{(\prime)}}$  in a doubly Cabibbo-suppressed way. Therefore the effective lifetimes turn out to be very robust with respect to the hadronic corrections, in analogy to the situation in  $B_s^0 \rightarrow J/\psi f_0$ . As in Ref. [6], we use  $\gamma = (68 \pm 7)^\circ$  in order to illustrate the hadronic effects. As far as the hadronic parameters are concerned, we consider the ranges

$$0 \leq b_{\eta^{(\prime)}} \leq 0.5, \quad 90^\circ \leq \vartheta_{\eta^{(\prime)}} \leq 270^\circ. \quad (34)$$

Due to the factor  $R_b \sim 0.5$  in (15) and (17) the range for  $b_{\eta^{(\prime)}}$  is conservative. The range for the strong phase is motivated by the topological structure of (15) and (17). It is also supported by an  $SU(3)_F$  analysis of  $B_d^0 \rightarrow J/\psi \pi^0$  data, where the counterparts of  $b_{\eta^{(\prime)}}$  and  $\vartheta_{\eta^{(\prime)}}$  in  $B_d^0 \rightarrow J/\psi K^0$ ,  $a$  and  $\theta$ , are found to be  $a \in [0.15, 0.67]$  and  $\theta \in [174, 212]^\circ$  at the  $1\sigma$  level [19] (see also Ref. [20]). For the ranges given in (34) the hadronic phase shift takes values in the interval

$$\Delta\phi_{J/\psi \eta^{(\prime)}} \in [-3^\circ, 0^\circ]. \quad (35)$$

Likewise, the direct CP asymmetry satisfies  $|C_{J/\psi \eta^{(\prime)}}| \lesssim 0.05$  under these assumptions and thereby has a negligible impact on  $\mathcal{A}_{\Delta\Gamma}^{J/\psi \eta^{(\prime)}}$ .

Once the  $B_s^0 \rightarrow J/\psi \eta^{(\prime)}$  effective lifetimes have been measured they can be converted into contours in the  $\phi_s$ - $\Delta\Gamma_s$  plane, as was pointed out in Ref. [21]. The interesting feature



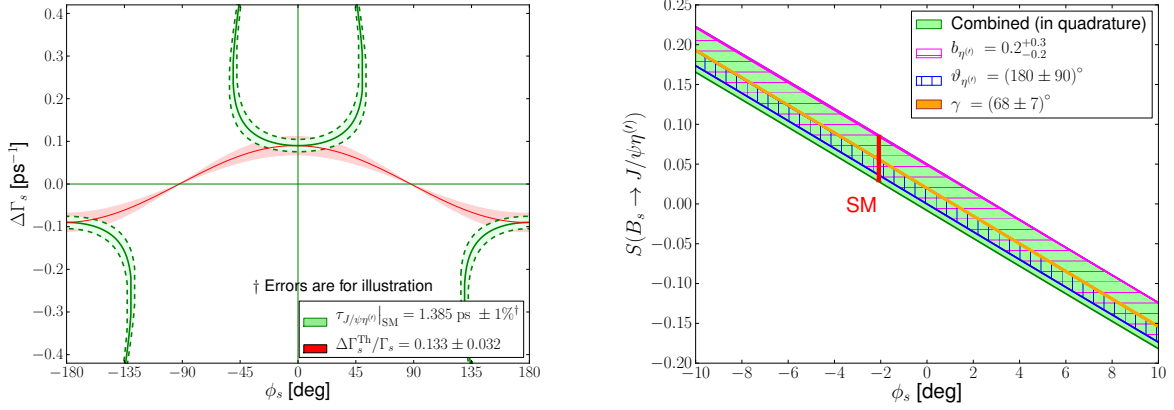


Figure 1: *Left panel:* the  $B_s^0 \rightarrow J/\psi\eta^{(l)}$  effective lifetime as a constraint in the  $\phi_s$ – $\Delta\Gamma_s$  plane. For illustration we have chosen a central value compatible with the SM values of  $\phi_s$  and  $\Delta\Gamma_s$  given in (32) and (37), respectively, and have assumed a lifetime measurement with 1% uncertainty. *Right panel:* the mixing-induced CP asymmetry of  $B_s^0 \rightarrow J/\psi\eta^{(l)}$  as a function of  $\phi_s$ , assuming  $\gamma = (68 \pm 7)^\circ$ ,  $0 \leq b_{\eta^{(l)}} \leq 0.5$  and  $90^\circ \leq \vartheta_{\eta^{(l)}} \leq 270^\circ$  for the calculation of the error band. We show only the region close to the SM case.

of this analysis is that it does not rely on the theoretical value  $\Delta\Gamma_s^{\text{Th}}$ , in contrast to the lifetime analysis given in Ref. [6]. Furthermore, using complementary information from a second CP-odd final state, such as  $B_s^0 \rightarrow J/\psi f_0$ , the mixing parameters  $\phi_s$  and  $\Delta\Gamma_s$  can be extracted. These can then be compared with information from other analyses, such as  $B_s^0 \rightarrow J/\psi\phi$ . In the left panel of Fig. 1, we show for illustration the lifetime contour that is compatible with the values of  $\phi_s$  and  $\Delta\Gamma_s$  given in (32) and (29), respectively. The corresponding theoretical SM prediction for the effective lifetimes is

$$\tau_{J/\psi\eta^{(l)}}|_{\text{SM}} = (1.385 \pm 0.029) \text{ ps}, \quad (36)$$

where we have used  $\tau_{B_s} = (1.477^{+0.021}_{-0.022}) \text{ ps}$  [22]. In the same plot we have also included a contour that arises from the plausible assumption that NP affects  $\Delta\Gamma_s$  only through  $B_s^0$ – $\bar{B}_s^0$  mixing [23], implying the relation

$$y_s = \frac{\Delta\Gamma_s^{\text{Th}} \cos \tilde{\phi}_s}{2\Gamma_s} = y_s^{\text{Th}} \cos(\tilde{\phi}_s^{\text{SM}} + \phi_s^{\text{NP}}). \quad (37)$$

Here  $\phi_s^{\text{NP}}$  is the NP  $B_s^0$ – $\bar{B}_s^0$  mixing phase, which also enters the phase  $\phi_s$  defined in (31) on which  $\mathcal{A}_{\Delta\Gamma}^{J/\psi\eta^{(l)}}$  depends, whereas the SM piece is given by  $\tilde{\phi}_s^{\text{SM}} = (0.22 \pm 0.06)^\circ$  [16].

There is an interesting trend of the current Tevatron and LHCb data to favour a value of  $\Delta\Gamma_s$  larger than (29), which raises the question of whether the corresponding theoretical analysis of  $\Delta\Gamma_s^{\text{Th}}$  fully includes all hadronic long-distance contributions [21]. It will be interesting to see if this trend will persist with future data or if it will eventually disappear.

A tagged analysis of  $B_s^0 \rightarrow J/\psi\eta^{(l)}$  decays allows the measurement of the following

CP-violating rate asymmetry:

$$\frac{\Gamma(B_s(t) \rightarrow J/\psi\eta^{(\prime)}) - \Gamma(\bar{B}_s(t) \rightarrow J/\psi\eta^{(\prime)})}{\Gamma(B_s(t) \rightarrow J/\psi\eta^{(\prime)}) + \Gamma(\bar{B}_s(t) \rightarrow J/\psi\eta^{(\prime)})} = \frac{C_{J/\psi\eta^{(\prime)}} \cos(\Delta M_s t) - S_{J/\psi\eta^{(\prime)}} \sin(\Delta M_s t)}{\cosh(\Delta\Gamma_s t/2) + \mathcal{A}_{\Delta\Gamma}^{J/\psi\eta^{(\prime)}} \sinh(\Delta\Gamma_s t/2)}, \quad (38)$$

where  $C_{J/\psi\eta^{(\prime)}}$  is the direct CP asymmetry that we have already encountered in (30), and

$$S_{J/\psi\eta^{(\prime)}} = -\sqrt{1 - C_{J/\psi\eta^{(\prime)}}^2} \sin(\phi_s + \Delta\phi_{J/\psi\eta^{(\prime)}}) \quad (39)$$

describes mixing-induced CP violation. The minus sign differs from the mixing-induced CP asymmetry of the  $B_s^0 \rightarrow J/\psi f_0$  channel [6] because of the opposite CP eigenvalues of the final states. In the right panel of Fig. 1, we show the dependence of  $S_{J/\psi\eta^{(\prime)}}$  on the mixing phase  $\phi_s$  and illustrate how the hadronic SM uncertainties as well as the uncertainties on  $\gamma$  propagate through. We observe that a future measurement of the mixing-induced CP asymmetry in the range

$$0.03 \lesssim S_{J/\psi\eta^{(\prime)}} \lesssim 0.09 \quad (40)$$

would not allow us to distinguish the SM from CP-violating NP contributions to  $B_s^0$ - $\bar{B}_s^0$  mixing. Should we encounter such a situation, more information would be required to accomplish this task. In this respect, things are similar to analyses of CP violation in  $B_s^0 \rightarrow J/\psi f_0$  [6] and  $B_s^0 \rightarrow J/\psi\phi$  [24]. In the case of the  $B_s^0 \rightarrow J/\psi\eta^{(\prime)}$  decays, the hadronic uncertainties can be controlled with the help of the  $B_d^0 \rightarrow J/\psi\eta^{(\prime)}$  channels.

## 4 The $B_d^0 \rightarrow J/\psi\eta^{(\prime)}$ Control Channels

The leading contributions to the  $B_d^0 \rightarrow J/\psi\eta$  decay originate from  $\bar{b} \rightarrow \bar{c}c\bar{d}$  quark-level processes. It is the formal counterpart of the  $B_d^0 \rightarrow J/\psi f_0$  mode discussed in Ref. [6]. Following this discussion, we write

$$A(B_d^0 \rightarrow J/\psi\eta) = -\lambda \mathcal{A}'_\eta \left[ 1 - b'_\eta e^{i\vartheta'_\eta} e^{i\gamma} \right] \quad (41)$$

with

$$\mathcal{A}'_\eta = -\lambda^2 A \left[ \sin\phi_P \left\{ \tilde{A}_E^{(c)} + \tilde{A}_{PA}^{(ct)} \right\} - \frac{1}{\sqrt{2}} \cos\phi_P \left\{ \tilde{A}_T^{(c)} + \tilde{A}_P^{(ct)} + 2\tilde{A}_E^{(c)} + 2\tilde{A}_{PA}^{(ct)} \right\} \right] \quad (42)$$

and

$$b'_\eta e^{i\vartheta'_\eta} = R_b \left[ \frac{\sin\phi_P \left\{ \tilde{A}_{PA}^{(ut)} \right\} - \frac{1}{\sqrt{2}} \cos\phi_P \left\{ \tilde{A}_P^{(ut)} + \tilde{A}_E^{(u)} + 2\tilde{A}_{PA}^{(ut)} \right\}}{\sin\phi_P \left\{ \tilde{A}_E^{(c)} + \tilde{A}_{PA}^{(ct)} \right\} - \frac{1}{\sqrt{2}} \cos\phi_P \left\{ \tilde{A}_T^{(c)} + \tilde{A}_P^{(ct)} + 2\tilde{A}_E^{(c)} + 2\tilde{A}_{PA}^{(ct)} \right\}} \right], \quad (43)$$

where we have used  $SU(3)_F$  arguments to identify the topological amplitudes with those in (11) and (12). The simplified expressions in (6) yield

$$\mathcal{A}'_\eta \approx \lambda^2 A \sqrt{\frac{1}{3}} \left[ \tilde{A}_T^{(c)} + \tilde{A}_P^{(ct)} + \tilde{A}_E^{(c)} + \tilde{A}_{PA}^{(ct)} \right], \quad (44)$$

$$b'_\eta e^{i\vartheta'_\eta} \approx R_b \left[ \frac{\tilde{A}_P^{(ut)} + \tilde{A}_E^{(u)} + \tilde{A}_{PA}^{(ut)}}{\tilde{A}_T^{(c)} + \tilde{A}_P^{(ct)} + \tilde{A}_E^{(c)} + \tilde{A}_{PA}^{(ct)}} \right]. \quad (45)$$

The key difference of the  $B_d^0 \rightarrow J/\psi\eta$  decay with respect to its  $B_s^0 \rightarrow J/\psi\eta$  counterpart is that the hadronic parameter  $b'_\eta e^{i\vartheta'_\eta}$  does not enter (41) in a doubly Cabibbo-suppressed way. Consequently, its impact is “magnified” in the  $B_d^0 \rightarrow J/\psi\eta$  observables. On the other hand, the branching ratio does suffer from a  $\lambda^2$  suppression.<sup>1</sup>

As discussed in detail in Ref. [6], the exchange and penguin amplitudes play a minor role and can be probed through the  $B_d^0 \rightarrow J/\psi\phi$  and  $B_s^0 \rightarrow J/\psi\pi^0$  decays, where already the currently available upper bound on the branching ratio of the former decay allows us to put the upper bound

$$\left| \frac{\tilde{A}_E^{(c)} + \tilde{A}_{PA}^{(ct)}}{\tilde{A}_T^{(c)}} \right| \lesssim 0.1. \quad (46)$$

Neglecting these contributions and using the  $SU(3)_F$  symmetry (as we have already implicitly done in the expression given above), we obtain

$$b_\eta e^{i\vartheta_\eta} \stackrel{SU(3)_F}{=} R_b \left[ \frac{\tilde{A}_P^{(ut)}}{\tilde{A}_T^{(c)} + \tilde{A}_P^{(ct)}} \right] \stackrel{SU(3)_F}{=} b'_\eta e^{i\vartheta'_\eta}. \quad (47)$$

Interestingly, the dependence on  $\phi_P$  drops out if the exchange and penguin annihilation contributions are neglected. Since the parameters  $b'_\eta$  and  $\vartheta'_\eta$  can be determined from the  $B_d^0 \rightarrow J/\psi\eta$  observables in a clean way (in analogy to the discussion for  $B_d^0 \rightarrow J/\psi f_0$  in Ref. [6]), we can control the penguin effects in the  $B_s^0 \rightarrow J/\psi\eta$  observables.

As the  $b_\eta^{(\prime)} e^{i\vartheta_\eta^{(\prime)}}$  are ratios of hadronic amplitudes, we expect (47) to be robust with respect to  $SU(3)_F$ -breaking corrections. Should the  $B_d^0 \rightarrow J/\psi\eta$  data favour a small value of  $b'_\eta$ , the exchange and penguin annihilation amplitudes could contribute significant uncertainties in relating  $b'_\eta$  to  $b_\eta$ . However, the doubly Cabibbo-suppressed corrections to the mixing-induced CP violation in  $B_s^0 \rightarrow J/\psi\eta$  would then be tiny anyway.

The  $B_d^0 \rightarrow J/\psi\eta$  decay was observed by the Belle collaboration [25], with

$$\text{BR}(B_d^0 \rightarrow J/\psi\eta) = [9.5 \pm 1.7(\text{stat.}) \pm 0.8(\text{syst.})] \times 10^{-6}, \quad (48)$$

which is consistent with the estimates given in Ref. [3]. We can use this measurement to obtain a first constraint on the hadronic parameters with the help of

$$H_\eta \equiv \frac{1}{\epsilon} \left| \frac{\mathcal{A}_\eta}{\mathcal{A}'_\eta} \right|^2 \left( \frac{M_{B_s^0} \Phi_s^\eta}{M_{B_d^0} \Phi_d^\eta} \right)^3 \frac{\tau_{B_s^0}}{\tau_{B_d^0}} \frac{\text{BR}(B_d^0 \rightarrow J/\psi\eta)}{\text{BR}(B_s^0 \rightarrow J/\psi\eta)}, \quad (49)$$

where the branching ratios refer to CP-averaged combinations. The formulae given above yield the following expression in terms of  $\gamma$  and the hadronic parameters:

$$H_\eta = \frac{1 - 2b'_\eta \cos \vartheta'_\eta \cos \gamma + b_\eta^2}{1 + 2\epsilon b_\eta \cos \vartheta_\eta \cos \gamma + \epsilon^2 b_\eta^2}. \quad (50)$$

---

<sup>1</sup>Analogous features apply to  $B_s^0 \rightarrow J/\psi K_S$  [4, 5],  $B_d^0 \rightarrow J/\psi f_0$  [6], and  $B_d^0 \rightarrow J/\psi\pi^0$  [19, 20].

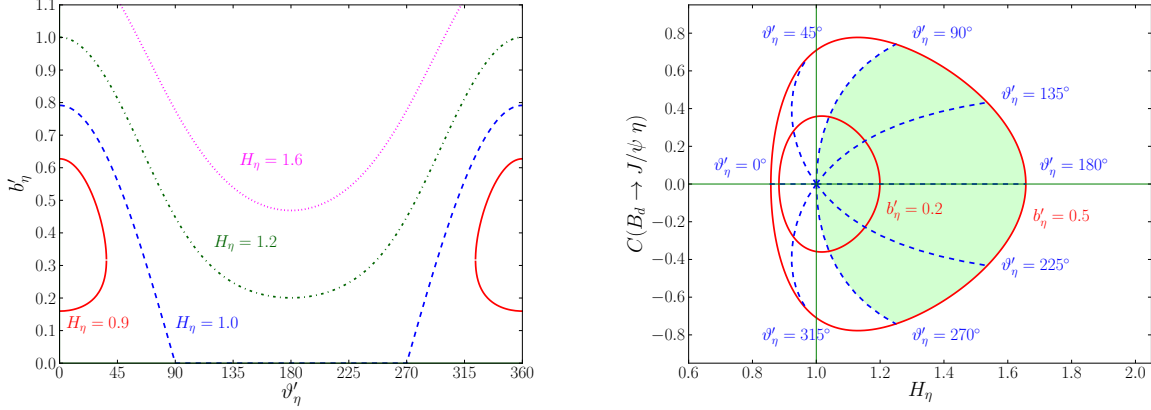


Figure 2: *Left panel:* constraints in the  $\vartheta'_\eta$ - $b'_\eta$  plane for various values of  $H_\eta$ . *Right panel:* correlation between  $H_\eta$  and the direct CP asymmetry of  $B_d^0 \rightarrow J/\psi \eta$ , where the solid rings correspond to  $b'_\eta = 0.2$  and  $0.5$  with  $\vartheta'_\eta$  allowed to vary; likewise, the dashed lines are fixed points of  $\vartheta'_\eta$  with  $b'_\eta$  allowed to vary. In both plots, we have assumed  $\gamma = 68^\circ$ .

In order to extract  $H_\eta$  from the branching ratios, we have to calculate the  $SU(3)$ -breaking ratio of the  $\mathcal{A}_\eta$  and  $\mathcal{A}'_\eta$  amplitudes. Using the factorization approximation and keeping only the leading tree contributions gives

$$\left| \frac{\mathcal{A}_\eta}{\mathcal{A}'_\eta} \right|_{\text{fact.}} = -\sqrt{2} \tan \phi_P \left[ \frac{F_1^{B_d^0 K^0}(M_{J/\psi}^2)}{F_1^{B_d^0 \pi^-}(M_{J/\psi}^2)} \right], \quad (51)$$

where we have – as in (23) – also neglected  $SU(3)_F$ -breaking corrections that originate from the down and strange spectator quarks. Using the form factors

$$F_1^{B_d^0 K^0}(M_{J/\psi}^2) = 0.615 \pm 0.076, \quad F_1^{B_d^0 \pi^-}(M_{J/\psi}^2) = 0.49 \pm 0.06 \quad (52)$$

calculated with the leading-order light-cone QCD sum-rule results of Ref. [26], as well as the measured  $B_{s,d}^0 \rightarrow J/\psi \eta$  branching ratios given earlier, we finally arrive at

$$H_\eta \times \left[ \frac{\tan 40^\circ}{\tan \phi_P} \right]^2 = 1.28_{-0.39}^{+0.61} \Big|_{\text{BR}}^{+0.50}_{-0.40} \Big|_{\text{FF}} = 1.28_{-0.56}^{+0.79}. \quad (53)$$

The errors reflect only the experimental and form-factor uncertainties and do not take non-factorizable  $SU(3)$ -breaking corrections into account. Using the factorization tests in (25), a future more precise measurement of the  $B_s^0 \rightarrow J/\psi \eta$  branching ratio should give us better quantitative insights into these effects.<sup>2</sup> In Fig. 2, we convert this result into contours in the  $\vartheta'_\eta$ - $b'_\eta$  plane (see Ref. [6] for details).

As soon as measurements of the CP asymmetries for  $B_d^0 \rightarrow J/\psi \eta$  become available we will be able to determine  $b'_\eta$  and  $\theta'_\eta$  in a clean way. Subsequently, we can determine  $H_\eta$

<sup>2</sup>Recent studies of other  $B_{(s)}$ -meson decays indicate small effects of this kind [14, 27].

through (50). Using then the information from the branching ratios and (49) and (51), we can determine  $|\tan \phi_P|$ . Alternatively, assuming that we will have a sharp picture of  $\phi_P$  by the time these measurements become available (see also Section 5), we can perform another test of non-factorizable  $SU(3)_F$ -breaking corrections.

The counterparts of the hadronic parameters in (16) and (17) for  $B_d^0 \rightarrow J/\psi\eta'$  are

$$\mathcal{A}'_{\eta'} \approx \lambda^2 A \sqrt{\frac{1}{6}} \left[ \tilde{A}_T^{(c)} + \tilde{A}_P^{(ct)} + 4\tilde{A}_E^{(c)} + 4\tilde{A}_{PA}^{(ct)} + \sqrt{6} \left( \tilde{A}_{E,gg}^{(c)} + \tilde{A}_{PA,gg}^{(ct)} \right) \tan \phi_G \right] \cos \phi_G \quad (54)$$

and

$$b'_{\eta'} e^{i\vartheta'_{\eta'}} \approx R_b \left[ \frac{\tilde{A}_P^{(ut)} + \tilde{A}_E^{(u)} + 4\tilde{A}_{PA}^{(ut)} + \sqrt{6} \tilde{A}_{PA,gg}^{(ut)} \tan \phi_G}{\tilde{A}_T^{(c)} + \tilde{A}_P^{(ct)} + 4\tilde{A}_E^{(c)} + 4\tilde{A}_{PA}^{(ct)} + \sqrt{6} \left( \tilde{A}_{E,gg}^{(c)} + \tilde{A}_{PA,gg}^{(ct)} \right) \tan \phi_G} \right], \quad (55)$$

respectively. Neglecting the exchange and penguin annihilation topologies, the control of the hadronic parameters in the  $B_s^0 \rightarrow J/\psi\eta'$  observables by means of the  $B_d^0 \rightarrow J/\psi\eta'$  mode is analogous to the case of the  $B_{s,d}^0 \rightarrow J/\psi\eta$  channels.

## 5 Determination of the $\eta$ - $\eta'$ Mixing Parameters

Let us finally discuss determinations of the  $\eta$ - $\eta'$  mixing parameters through measurements of the  $B_{s,d}^0 \rightarrow J/\psi\eta^{(\prime)}$  branching ratios. If we project out on the singlet states in (4) and (5) and assume that the exchange and penguin annihilation topologies give negligible contributions, we obtain the relation

$$R_s \equiv \frac{\text{BR}(B_s^0 \rightarrow J/\psi\eta')}{\text{BR}(B_s^0 \rightarrow J/\psi\eta)} \left( \frac{\Phi_s^\eta}{\Phi_s^{\eta'}} \right)^3 = \frac{\cos^2 \phi_G}{\tan^2 \phi_P} = 1.3_{-0.5}^{+1.5}, \quad (56)$$

which does not assume  $SU(3)_F$  or factorization; the numerical value corresponds to the Belle result [12]. The same expression with  $\phi_G = 0$  has already been given in Ref. [28].

In analogy to (56), we introduce the following ratio for the  $B_d$  decays:

$$R_d \equiv \frac{\text{BR}(B_d^0 \rightarrow J/\psi\eta')}{\text{BR}(B_d^0 \rightarrow J/\psi\eta)} \left( \frac{\Phi_d^\eta}{\Phi_d^{\eta'}} \right)^3 = \cos^2 \phi_G \tan^2 \phi_P. \quad (57)$$

Using the experimental value in (48), this expression results in the prediction

$$\text{BR}(B_d^0 \rightarrow J/\psi\eta') = \left[ \frac{\cos \phi_G}{\cos 20^\circ} \right]^2 \left[ \frac{\tan \phi_P}{\tan 40^\circ} \right]^2 \times (4.7 \pm 0.9) \times 10^{-6}. \quad (58)$$

Only the upper bound  $\text{BR}(B_d^0 \rightarrow J/\psi\eta') < 6.3 \times 10^{-5}$  (90% C.L.) is currently available from the BaBar collaboration [29].

Once the  $B_d^0 \rightarrow J/\psi\eta'$  branching ratio has been measured, we can use

$$\frac{R_d}{R_s} = \tan^4 \phi_P, \quad R_s R_d = \cos^4 \phi_G \quad (59)$$

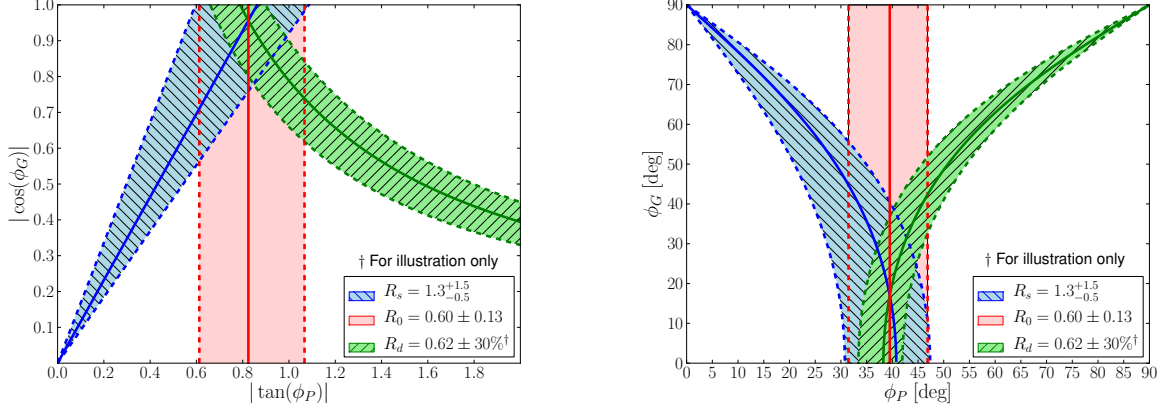


Figure 3: Constraints on the  $\eta$ - $\eta'$  mixing parameters from the  $B_{s,d}^0 \rightarrow J/\psi\eta^{(\prime)}$  and  $B_d^0 \rightarrow J/\psi\pi^0$  branching ratios as discussed in the text. Note that the right panel does not show all the discrete angular ambiguities.

to determine the mixing angles up to fourfold discrete ambiguities. It is interesting to note that the 4th powers in these expressions result in precise determinations of  $|\tan \phi_P|$  and  $|\cos \phi_G|$  even for branching ratio measurements with significant errors. If we assume, for illustration, future measurements of  $R_s = 1.3 \pm 0.4$  and  $R_d = 0.6 \pm 0.2$ , i.e. with precisions of 30%, we would obtain  $\phi_P = (39.5 \pm 3.1)^\circ$  and  $|\phi_G| \in [0^\circ, 33^\circ]$ .

In Fig. 3, we have illustrated this method, showing the contours for the current experimental value of  $R_s$  in (56) and our illustrative value of  $R_d = 0.6 \pm 0.2$ . It is interesting to include also the constraint from the following ratio [30]:

$$R_0 \equiv \frac{\text{BR}(B_d^0 \rightarrow J/\psi\eta)}{\text{BR}(B_d^0 \rightarrow J/\psi\pi^0)} \left( \frac{\Phi_d^{\pi^0}}{\Phi_d^\eta} \right)^3 = \cos^2 \phi_P. \quad (60)$$

Here penguin annihilation and exchange topologies were again neglected. The penguin parameters of the  $B_d^0 \rightarrow J/\psi\pi^0$  decay [19, 20] are then the same as in the  $B_d^0 \rightarrow J/\psi\eta^{(\prime)}$  modes. In particular, we expect also the same direct and mixing-induced CP asymmetries. As (60) does not depend on  $\phi_G$ , we can straightforwardly convert the Belle result in (48) with  $\text{BR}(B_d^0 \rightarrow J/\psi\pi^0) = (1.76 \pm 0.16) \times 10^{-5}$  [7] into

$$\phi_P|_{R_0} = (40^{+7}_{-8})^\circ. \quad (61)$$

The intersection of the corresponding band in Fig. 3 with the  $R_s$  contour gives  $\phi_G = 17^\circ$  for the central values. These results are in good agreement with those discussed at the beginning of Section 2.

## 6 Conclusions

The  $B_s^0 \rightarrow J/\psi\eta^{(\prime)}$  decays offer interesting insights into the  $B_s^0$ - $\bar{B}_s^0$  mixing parameters through their effective lifetimes and mixing-induced CP asymmetries. We have performed an analysis of these observables, focusing on hadronic SM corrections which

enter in a doubly Cabibbo-suppressed way. It turns out that the effective lifetimes are particularly robust with respect to these effects. Once they have been measured, we can convert the corresponding experimental results into contours in the  $\phi_s - \Delta\Gamma_s$  plane. As far as the mixing-induced CP asymmetries are concerned, measured values within the range  $0.03 \lesssim S_{J/\psi\eta^{(\prime)}} \lesssim 0.09$  would not allow us to distinguish CP-violating NP contributions to  $B_s^0 - \bar{B}_s^0$  mixing from SM effects, unless we can control the hadronic SM corrections.

We have shown that this can be accomplished with the help of the  $B_d^0 \rightarrow J/\psi\eta^{(\prime)}$  channels and the  $SU(3)_F$  flavour symmetry. In these decays, the relevant hadronic parameters are not doubly Cabibbo-suppressed. Only a branching ratio measurement for  $B_d^0 \rightarrow J/\psi\eta$  is available from the Belle collaboration, which we have used to obtain the first value of the  $H_\eta$  observable. This observable implies (still pretty poor) constraints for the hadronic parameters. The next important step to constrain them in a more stringent way would be the measurement of direct CP violation in  $B_d^0 \rightarrow J/\psi\eta$ . Other interesting control channels in this respect are  $B_s^0 \rightarrow J/\psi K_S$  and  $B_d^0 \rightarrow J/\psi\pi^0$ . If exchange and penguin annihilation topologies are neglected, they depend on the same penguin parameters  $b'_{\eta^{(\prime)}}$  and  $\theta'_{\eta^{(\prime)}}$ . It is important to obtain stronger experimental constraints on these topologies in the future through decays such as  $B_d^0 \rightarrow J/\psi\phi$  and  $B_s^0 \rightarrow J/\psi\pi^0$ .

In addition to exploring CP violation, the  $B_s^0 \rightarrow J/\psi\eta^{(\prime)}$  and  $B_d^0 \rightarrow J/\psi\eta^{(\prime)}$  decays allow us to probe non-factorizable  $SU(3)_F$ -breaking effects and offer interesting strategies for determining the  $\eta$ - $\eta'$  mixing parameters from their ratios of branching ratios,  $R_s$  and  $R_d$ . Unfortunately, the  $B_d^0 \rightarrow J/\psi\eta'$  branching ratio, which we predict at the  $5 \times 10^{-6}$  level, has not yet been measured. But using the other currently available  $B_{s,d}^0 \rightarrow J/\psi\eta^{(\prime)}$  data in combinations with  $\text{BR}(B_d^0 \rightarrow J/\psi\pi^0)$ , we obtain a picture for the mixing angles  $\phi_P$  and  $\phi_G$  in good agreement with other information. Future measurements of  $R_s$  and  $R_d$  with 30% precision would result in uncertainties of  $\Delta\phi_P \sim \pm 3^\circ$  and  $\Delta\phi_G \sim \pm 15^\circ$ .

We have seen that the amplitude structures of the  $B_{s,d}^0 \rightarrow J/\psi\eta$  and  $B_{s,d}^0 \rightarrow J/\psi\eta'$  decays correspond formally to the quark-antiquark and tetraquark descriptions of the  $f_0$  in  $B_{s,d}^0 \rightarrow J/\psi f_0$ , respectively. From the theoretical point of view, the situation in the  $B_{s,d}^0 \rightarrow J/\psi\eta^{(\prime)}$  system is more favourable than in  $B_{s,d}^0 \rightarrow J/\psi f_0$  as the hadronic composition of the  $f_0$  is still not settled. On the other hand, the latter system is more promising from an experimental point of view because of the dominant  $f_0 \rightarrow \pi^+\pi^-$  channel. The most prominent  $\eta^{(\prime)}$  decays involve photons or neutral pions in the final states, which is a very challenging signature for  $B$ -decay experiments at hadron colliders and appears better suited for the future  $e^+e^-$  SuperKEKB and SuperB projects, which is also reflected by the previous Belle and BaBar analyses of the  $B_{d,s}^0 \rightarrow J/\psi\eta^{(\prime)}$  modes. We hope that our experimental colleagues will eventually meet the practical challenges, thereby putting yet another system on the roadmap for testing the CP-violating sector of the SM, probing non-factorizable  $SU(3)_F$ -breaking effects and exploring  $\eta$ - $\eta'$  mixing.

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